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Multichannel oscillations and relations between LSND, KARMEN and MiniBooNE, with and without CP violation*

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We show by examples that multichannel mixing can affect both the parameters extracted from neutrino oscillation experiments, and that more general conclusions derived by fitting the experimental data under the assumption that only two channels are involved in the mixing. Implications for MiniBooNE are noted and an example based on maximal CP violation displays profound implications for the two data sets (ν_μ and $\bar{\nu_\mu}$) of that experiment.

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I. INTRODUCTION

There has been much discussion[1, 2] concerning the difference in results between the KARMEN[3] and LSND[4] experiments regarding appearance of electron antineutrinos, $\bar{\nu}_e$, from muon antineutrino, $\bar{\nu}_\mu$, sources. Initially, the concern was that the mass-squared difference, Δm^2, characterizing the oscillation scale does not match up with the differences observed in atmospheric[5] and solar[6] neutrino oscillations (with the latter now both confirmed and superseded by the results of the KamLAND[7] experiment). This concern was based on the assumption that only the three known active neutrino flavors participate in the oscillations, so that the third value of Δm^2 is determined by the other two values.

More recently, it has been recognized that light sterile neutrinos may exist and participate in oscillation phenomena[8, 9, 10]. Because multiple cycles have not been explicitly observed, this raises a serious question regarding an assumption in the analyses of all oscillation experiments to date, namely that the oscillation scales are sufficiently separated so as not to influence the the values extracted using the functional relations in simple, two-channel mixing. We have previously shown[8] how a reduced rank see-saw[11, 12] which couples several different oscillations, leads to more complex phenomena, as was long ago recognized by Fermi and Ulam[13] whenever more than two oscillators are coupled.

If there are, in fact, multiple paths which contribute to electron neutrino appearance from a muon neutrino source, then the shortest baseline (largest Δm^2) oscillation would appear as excursions from a rising longer baseline[8]. The other paths would have independent oscillation parameters Δm^2_{1i} and Δm^2_{2j}, where, without loss of generality, we treat mass eigenstate "1" is the dominant contributor to the initial flavor and "2" as the dominant contributor to the final flavor. Then the oscillation from initial to final flavor can be represented as

$$\begin{aligned} P(\bar{\nu}_\mu \to \bar{\nu}_e) &= A^2 \sin^2(\Delta m^2_{12} x) \\ &+ B^2 \sin^2(\Delta m^2_{1i} x) \\ &+ C^2 \sin^2(\Delta m^2_{2i} x) + \cdots \end{aligned} \tag{1}$$

where $x = 1.27\ L/E$ (m/MeV) and the dots indicate that, in principle, more than one intermediate channel i may contribute. The coefficients must all be positive semi-definite to ensure positivity of the appearance probability. The additional terms produce the rising baseline.[8] so that the problem may be viewed as an oscillation of the usual two channel type, but occurring over a rising baseline. Of course, for x sufficiently small, all of the $\sin^2$ arguments can be simultaneously expanded and the appearance probability develops purely quadratically, viz.

$$\begin{aligned} P(\bar{\nu}_\mu \to \bar{\nu}_e) &\sim \left[A^2(\Delta m^2_{12})^2 + B^2(\Delta m^2_{1i})^2\right. \\ &+ \left. C^2(\Delta m^2_{2i})^2 + \cdots\right] x^2 \end{aligned} \tag{2}$$

We provide some simple illustrations, which have the advantage of being able to improve the compatibility between the KARMEN and LSND results in a way that implicitly makes predictions for the eagerly awaited results from the MiniBooNE[16] experiment.

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II. EXAMPLE WITH CP CONSERVATION

We suppose that an oscillation from $\bar{\nu}_\mu$ to $\bar{\nu}_e$ occurs with a value of Δm^2 consistent with the LSND allowed range. However, as was shown possible in Ref.[8], we assume this occurs with coupling to other (here unspecified) channels that produces a rising baseline for the two channel oscillation. Thus, the probability for detecting a $\bar{\nu}_e$ of energy E at a distance L from a $\bar{\nu}_\mu$ source, $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ is given by the explicit part of Eq.(1) above, and here we choose some particular examples for the parameter values (where we absorb the factor of 1.27 into coefficients so that the values in the arguments of the sine functions are $1.27 \times \Delta m^2$ in units of eV^2.):

$$
\begin{aligned}
P^{(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)}_{\mathrm{2ch;High}} &= 0.0045 \sin^2(0.8\ L/E) && (3)\\
P^{(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)}_{\mathrm{2ch;Low}} &= 0.0600 \sin^2(0.2\ L/E) && (4)\\
P^{(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)}_{\mathrm{multich;a}} &= 0.005 \sin^2(0.7\ L/E) \\
&+ 0.001 \sin^2(0.3\ L/E) \\
&+ 0.0025 \sin^2(0.4\ L/E) && (5)\\
P^{(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)}_{\mathrm{multich;b}} &= 0.004 \sin^2(0.7\ L/E) \\
&+ 0.005 \sin^2(0.2\ L/E) \\
&+ 0.002 \sin^2(0.5\ L/E) && (6)
\end{aligned}
$$

for the appearance rates in the two-channel and multichannel cases respectively. Although the three-channel mass relation is satisfied in this example, we emphasize that the intermediate channel need not be an active neutrino (if light sterile neutrinos exist[9]) and furthermore, this relation need not have been satisfied if two different intermediate channels make the dominant contributions[1]. For very large Δm^2, rapid oscillations will average to a constant appearance rate independent of L/E, which we use to set a normalization of 0.0026 consistent with the scale for the signal reported by LSND[4].

For the multichannel cases, Fig. 1 reprises the character of the result in Fig.(2) of Ref.[8] in the usual L/E terms. Note that the appearance probabilities are virtually indistinguishable in the low L/E region covered by the KARMEN and LSND experiments, although wide deviations occur in the larger L/E region that MiniBooNE can address. It is also interesting to consider an E/L plot of the same function as in Eq.(1), along with the corresponding distributions for simple two-channel fits to the LSND experiment at high and low values of Δm^2 as shown in Fig. 2. Again, for a very high value of Δm^2, the limited resolution results in an averaged, flat distribution, experimentally indistinguishable from one independent of E/L.

Both figures include an indication of the range of L/E (or E/L) over which each of the KARMEN, LSND and MiniBooNE experiments are sensitive.

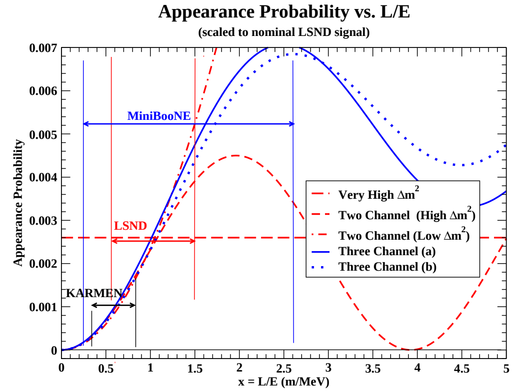

FIG. 1: Two channel mixing $\bar{\nu}_e$ appearance probabilities from $\bar{\nu}_\mu$ for three values of Δm^2 compared with the two example three-channel rates discussed in the text *vs.* L/E .

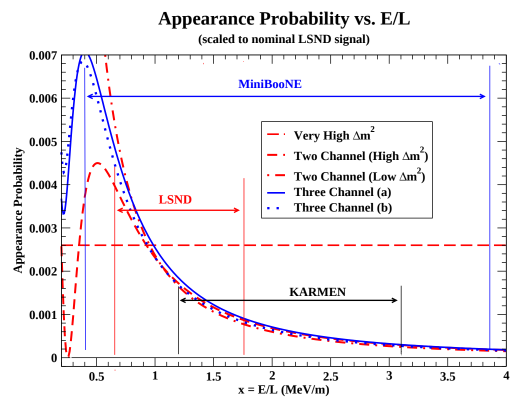

FIG. 2: Two channel mixing $\bar{\nu}_e$ appearance probabilities from $\bar{\nu}_\mu$ for three values of Δm^2 compared with the two example three-channel rates discussed in the text *vs.* E/L .

III. EXAMPLE WITH CP VIOLATION

In the discussion so far, we have not allowed for CP violation. If that occurs, additional terms[14] arise from the imaginary part of the same product of four mixing matrices that produces the positive coefficients above from the real part. These terms are of the form

$$
\begin{aligned}
\Delta P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) &= D \sin(2\Delta m^2_{12} x) + E \sin(2\Delta m^2_{1i} x) \\
&+ F \sin(2\Delta m^2_{2i} x) + \cdots && (7)
\end{aligned}
$$

Note that the constraint of positivity of the coefficients does not apply to these terms. In fact, for "maximal" CP violation, in the sense of the conventional angle $\delta = \pi/2$, it is straightforward to demonstrate that, depending on which way one represents the mass differ-

ences, some of the coefficients above must be negative semi-definite. Specifically, in the Particle Data Group (PDG) formulation[14] for exactly three channels

$$\begin{aligned} D &= -E = -F = s_{12}s_{23}s_{13}c_{23}c_{12}c_{13}^2 \\ A^2 &= c_{12}^2 s_{13}^2 s_{23}^2 c_{13}^2 \\ B^2 &= s_{12}^2 s_{13}^2 s_{23}^2 c_{13}^2 \\ C^2 &= s_{12}^2 c_{12}^2 c_{13}^2 (c_{23}^2 - s_{23}^2 s_{13}^2) \end{aligned} \quad (8)$$

where $s_{12} = \sin(\theta_{12})$, etc. as usual. Clearly, if any one of the conventional CKM/MNS[15] angles vanishes (or reaches $\pi/2$) then the CP-violating parts vanish as they must. Note also that the positivity of the $\sin^2$ terms is not affected.

It is the positivity of the appearance probability that requires the relation above between D, E and F – these terms all change sign for the CP-conjugate channel. Therefore, the sum of the coefficients of L/E for small L/E must actually vanish:

$$D \times \left[\Delta m_{12}^2 - \Delta m_{1i}^2 + \Delta m_{2i}^2\right] = 0 \quad (9)$$

which is guaranteed by the relations among the three mass differences.

We provide one example with CP violation of an oscillation that agrees with both LSND and KARMEN and predicts that the signal in MiniBooNE may be smaller than the largest value expected from the LSND results:

$$\begin{aligned} P_{\text{CPV}}^{(\bar\nu_\mu\to\bar\nu_e)} &= 0.0025\,(\sin^2(1.0\,L/E)) - 0.001\,\sin(2.0\,L/E) \\ &+\ 0.0005\,(\sin^2(3.0\,L/E)) - 0.001\,\sin(6.0\,L/E) \\ &+\ 0.001\,(\sin^2(4.0\,L/E)) + 0.001\,\sin(8.0\,L/E) \\ &+\ 0.01\,(\sin^2(0.5\,L/E)) \end{aligned} \quad (10)$$

where coefficients of all additional terms are assumed to be negligibly small (and we have again absorbed the factor of 1.27 into the numerical parameters). Of course, none of the Δm^2 values matches with those inferred from other experiments that do observe flavor oscillations, so the scenario here is viable *only* in the case that this set of oscillations is proceeding through neutrino mass eigenstates that are dominantly *sterile* neutrino states, with small flavor components.

For the opposite CP process, applicable to KARMEN and LSND, the contributions from the imaginary part of the product of the U matrices (the $\sin(2\Delta m^2 x)$ terms) change sign, so we have

$$\begin{aligned} P_{\text{CPV}}^{(\nu_\mu\to\nu_e)} &= 0.0025\,(\sin^2(1.0\,L/E)) + 0.001\,\sin(2.0\,L/E) \\ &+\ 0.0005\,(\sin^2(3.0\,L/E)) + 0.001\,\sin(6.0\,L/E) \\ &+\ 0.001\,(\sin^2(4.0\,L/E)) - 0.001\,\sin(8.0\,L/E) \\ &+\ 0.01\,(\sin^2(0.5\,L/E)) \end{aligned} \quad (11)$$

This formula improves the agreement between KARMEN and LSND, slightly, over the lowest Δm^2 fit to both, and

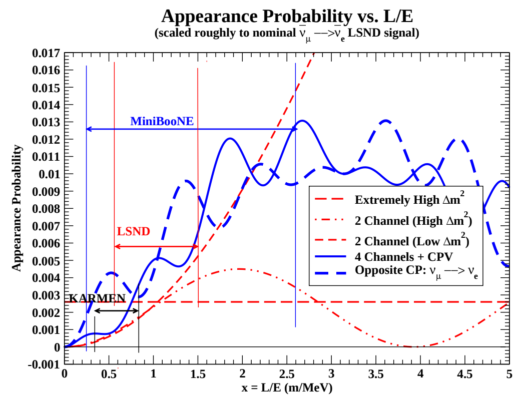

FIG. 3: Four channel mixing $\bar\nu_e$ appearance probability from $\bar\nu_\mu$ for Eq.(10) and the appearance probability for the CP conjugate channel for Eq.(11) as given in the text *vs.* L/E compared with 2 channel descriptions described previously.

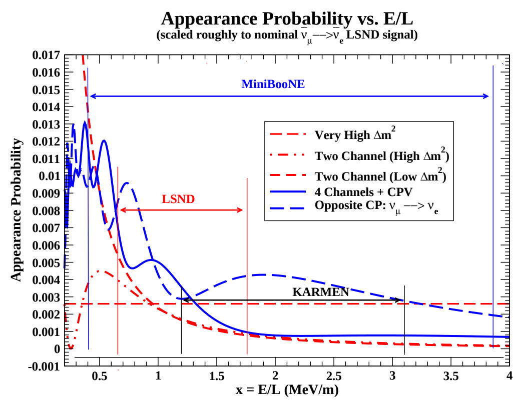

FIG. 4: Four channel mixing $\bar\nu_e$ appearance probability from $\bar\nu_\mu$ for Eq.(10) and the appearance probability for the CP conjugate channel for Eq.(11) as given in the text *vs.* E/L compared with 2 channel descriptions described previously.

does so without requiring large ν_μ or ν_e disappearance at larger L/E – the maximum loss is only slightly more than 1%.

We plot the formulae in Eqs.(10) and (11) vs. L/E and E/L in Figs. 3 and 4 for comparison with the CP-conserving oscillations shown earlier. As is apparent both from these Figures and from the crude Δm^2 values, these formulae do not represent a "fit" to the data, but are simply an indication of the possibilities available if one does not arbitrarily constrain the entire neutrino oscillation picture to three active Majorana neutrino flavor and (light) mass eigenstates.

IV. DISCUSSION

The form in Eq.(1) and the additional terms in Eq.(7) have far too many parameters to be tightly fit with data available from present neutrino experiments, hence the predeliction for fitting to two-channel mixing scenarios. (Even when more channels are attempted, the dominance of one scale has been assumed[5] or the two-channel fit results are used[1].) However, since the rising baseline we observed possible[8] is roughly quadratic, corresponding to the opening of contributions from a longer wavelength oscillation, it should be viable to include the possibility of a rising baseline with one additional parameter, T, viz.,

$$P(\bar{\nu}_{init} \to \bar{\nu}_{final}) \;\sim\; A^2 \sin^2(1.27\Delta m^2 L/E) + T(1.27\Delta m^2 L/E)^2 + \cdots \quad (12)$$

without CP violation, where T accounts for the rising baseline/additional channels, or even more compactly,

$$P(\bar{\nu}_{init} \to \bar{\nu}_{fin}) \;\sim\; S(1.27\Delta m^2 L/E)^2 + V(1.27\Delta m^2 L/E)^3 + \cdots \quad (13)$$

where S absorbs all of relative amplitudes and ratios of Δm^2 values of longer wavelength channels into one parameter and V absorbs all of the additional parameters for any number of CP-violating effects into another. In fact, these functional forms do not even depend on our initial physical ansatz , Eq.(1), and have only the disadvantage of not being applicable for large values of L/E. We recommend that all oscillation experiments test such functional forms to determine whether or not the χ-squared per degree of freedom of the fit to their data is, or is not, improved by such additions to the standard two-channel analysis.

As noted in Ref.[2], with two-channel mixing, the LSND and KARMEN experiments are in best agreement for low values of Δm^2 because in that case, the difference in distances affects the results quadratically in favor of LSND, whereas for very high values of Δm^2 they should have seen the same size signal and hence are in disagreement. However, here we see that, while even for an intermediate value of Δm^2 the agreement would be marginal in two-channel mixing, the problem is reduced once the effect of a third channel on the baseline for the two-channel oscillation is included. The improvement is even more striking when CP violation is allowed. In fact, the ratio between the signal expected in the two experiments can achieve essentially the same value (or an even better one) as that obtained with a small Δm^2 fit.

The concern over a smaller value of Δm^2 for LSND is the effect of the larger intrinsic mixing amplitude required to match the data obtained in the region of small L/E: It predicts large effects, particularly disappearance rates, at much larger values of L/E typical of reactor experiments[17], for instance. However, as our examples demonstrate, the rising baseline breaks the relation, seen in two-channel mixing, between the rate that an appearance signal increases with increasing L/E and the size of the signal in the initial range of the effect. As shown in our examples, the total appearance rate remains near 1% at all values of L/E, completely consistent with the limits from short baseline disappearance experiments[17, 18] for both ν_μ and ν_e. We emphasize that this is true even though a two-channel fit would require a much larger overall amplitude in order for the signal to have grown to the size reported by LSND yet have remained too small to be observed by KARMEN with its shorter baseline (which one would think not likely to be significantly shorter).

Finally, we note that the inclusion of CP violation, which is to be expected, (but not CPT violation, which would be revolutionary) further improves agreement between KARMEN and LSND and makes explicit testable predictions for the results of MiniBooNE, as long as the possibility of light sterile neutrinos is allowed. We reiterate that small amplitude mixing to light sterile neutrinos poses no conflict with any known laboratory experimental data.

We conclude that only when oscillation experiments can all provide unbiased L/E distributions, rather than reporting parameters for two-channel fits to the oscillations observed, will definitive conclusions be possible, regarding neutrino mass and mixing parameters, that are independent of theoretical biases.

The examples we have presented also suggest that, at low energy, the MiniBooNE experiment may observe a considerably larger *or* smaller signal for ν_e appearance than would be expected from the two-channel fits to KARMEN and LSND. However, the examples also show that this would *not* necessarily contradict the results of either of those two experiments, whether considered separately or jointly.

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